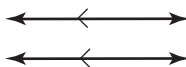


6.4

Parallel and Perpendicular Lines

parallel lines

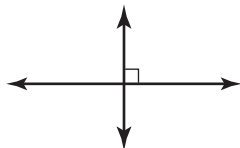
- lines that run in the same direction and never cross



- matching arrow symbols indicate that lines are parallel

perpendicular lines

- lines that intersect at right angles



- a small box at the intersection indicates a 90° angle



Tools

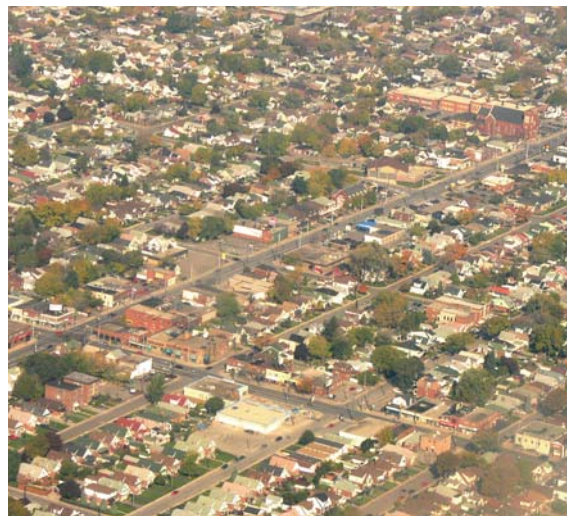
- grid paper
- protractor
- graphing calculator

Parallel and

perpendicular lines

appear all the time in the world around you. Think of streets, wallpaper, railway tracks, and skyscrapers. Look around your classroom. Where can you see examples of

- parallel lines?
- perpendicular lines?



Investigate

How are the slopes of parallel and perpendicular lines related?

- Graph each set of lines on the same grid.

a) $y = 2x - 3$

$y = 2x + 1$

$y = 2x - 2$

b) $y = -\frac{1}{2}x + 4$

$y = -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x - 1$

- How are the lines in each set in step 1 related geometrically? How are the equations related?

- Graph each pair of lines on the same grid. Use the same scale on both axes. If you are using a graphing calculator, press **ZOOM** and select **5:ZSquare**.

a) $y = 2x + 3$

b) $y = -\frac{2}{3}x + 3$

c) $y = -x + 1$

$y = -\frac{1}{2}x + 1$

$y = \frac{3}{2}x - 4$

$y = x - 3$

- How are the lines in each set in step 3 related geometrically? How are the equations related?

5. Reflect

- How can you tell whether lines are parallel from their equations?
- How can you tell whether lines are perpendicular from their equations?

Example Slopes of Parallel and Perpendicular Lines

- a) The equation of a line is $y = 3x - 4$. Give the slope of a parallel line.
- b) The equation of a line is $y = \frac{3}{5}x + 2$. Give the slope of a perpendicular line.

Solution

- a) The line $y = 3x - 4$ has slope 3.
A parallel line will have the same slope, 3.

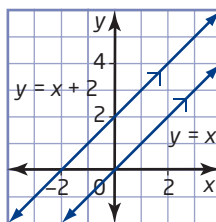
- b) The line $y = \frac{3}{5}x + 2$ has slope $\frac{3}{5}$.

A perpendicular line will have slope $-\frac{5}{3}$.

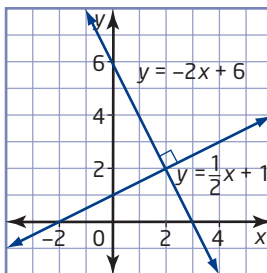
To find the perpendicular slope, I turn the fraction upside down and use the opposite sign.

Key Concepts

- The slopes of parallel lines are the same.



- The slopes of perpendicular lines are **negative reciprocals**.



negative reciprocals

- two numbers whose product is -1

Communicate Your Understanding

C1 Which lines are parallel? Explain how you know.

A $y = \frac{3}{4}x + 1$

B $y = 3x + 4$

C $y = \frac{3}{4}x$

D $y = \frac{4}{3}x + 1$

C2 Which lines are perpendicular? Explain how you know.

A $y = 3x + 2$

B $y = -\frac{1}{3}x - 5$

C $y = \frac{1}{3}x + 3$

D $y = -3x - 1$

Practise

1. Graph each pair of lines on the same coordinate grid. Find their slopes and conclude whether the lines are parallel, perpendicular, or neither.

a) $y = \frac{1}{4}x - 2$ $y = \frac{1}{4}x + 3$

b) $y = 2x + 5$ $4x - 2y + 6 = 0$

c) $x + y = 4$ $y = x - 3$

d) $y = \frac{1}{2}x - 4$ $x - 2y + 1 = 0$

2. Graph each pair of lines on the same coordinate grid. Find their slopes and conclude whether the lines are parallel, perpendicular, or neither.

a) $y = 3$ $x = -2$ b) $y = -4$ $y = x$

c) $x = 5$ $x = 0$ d) $y = x + 1$ $y = -x$

Connect and Apply

3. The slopes of two lines are given. Conclude whether the lines are parallel, perpendicular, or neither. Justify your answers.

a) $m = \frac{2}{3}$, $m = \frac{4}{6}$ b) $m = \frac{3}{4}$, $m = -\frac{4}{3}$

c) $m = 2$, $m = -2$ d) $m = 1$, $m = -1$

e) $m = \frac{1}{5}$, $m = 0.2$ f) $m = 2\frac{1}{4}$, $m = -\frac{4}{9}$

For help with questions 4 and 5, see the Example.

4. What is the slope of a line that is parallel to each line?

a) $y = \frac{3}{5}x - 2$ b) $y = -x + 7$

c) $2x - y + 3 = 0$ d) $4x + 3y = 12$

e) $y = 2$ f) $x = -5$

5. For each line in question 4, give the slope of a perpendicular line.

6. Write the equations of two lines that are parallel to the line $3x - 6y - 5 = 0$.

7. Write the equations of two lines that are perpendicular to the line $4x + y - 2 = 0$.

8. A triangle has vertices A(1, 2), B(3, 8), and C(6, 7).

- a) Plot these points and draw the triangle.
b) Does this appear to be a right triangle? Explain.
c) Find the slopes of the line segments that form this triangle.
d) Explain how the slopes can be used to conclude whether or not this is a right triangle. Is it?

9. Determine whether or not the following sets of points form right triangles. Justify your answers with mathematical reasoning.
- a) $A(1, 1)$, $B(-2, 5)$, $C(3, -2)$
 - b) $P(2, 4)$, $Q(-2, 2)$, $R(5, -2)$
10. $\triangle KLM$ has vertices $K(-2, 3)$ and $L(-6, -2)$.
- a) Find the coordinates of M such that $\triangle KLM$ is a right triangle.
 - b) Is there more than one solution? Explain.

Achievement Check

11. a) Using intercepts, graph the three lines $4x + y - 8 = 0$, $2x - y - 4 = 0$, and $x + 2y - 16 = 0$ on the same coordinate grid.
- b) The three lines form a triangle. Does this triangle appear to be a right triangle?
 - c) Using slopes, explain how you can be sure of your conclusion in part b).
 - d) Give the equations for three different lines that do form a right triangle.

Extend

12. a) Graph this pair of lines and identify their x - and y -intercepts.
- $$2x + 5y = 10 \qquad 2x + 5y = -10$$
- b) Repeat part a) for this pair of lines.
$$3x + 4y = 12 \qquad 3x + 4y = -12$$
 - c) Describe how you can use intercepts to quickly find a line that is parallel to a given line. Create an example of your own to support your explanation.
13. a) Graph this pair of lines and identify their x - and y -intercepts.
- $$3x + 5y = 15 \qquad 5x - 3y = -15$$
- b) Repeat part a) for this pair of lines.
$$2x + 7y = 14 \qquad 7x - 2y = -14$$
 - c) Describe how you can use intercepts to quickly find a line that is perpendicular to a given line. Create an example of your own to support your explanation.
14. **Math Contest** A and k are one-digit numbers. Given two lines, $Ax - 3y + 15 = 0$ and $y = kx + 7$, determine the number of pairs of values for A and k for which the two lines are
- a) parallel
 - b) perpendicular
 - c) coincident (the same line)